Grade 7/8 Math Circles

March 25th - 28th, 2024

Continued Fractions - Problem Set

1. Rewrite each rational number as the unique $\frac{a}{b}$ representation where a and b are both integers.
   (a) $\dfrac{1/2}{2}$
   (b) $\dfrac{5/7}{8/4}$
   (c) $\dfrac{4.3}{8.5}$
   (d) $\dfrac{2.3}{3.2}$
2. A little monkey had 60 peaches.
   On the **first** day he decided to keep $\frac{3}{4}$ of his peaches. He gave the rest away. Then he ate one.
   On the **second** day he decided to keep $\frac{7}{11}$ his peaches. He gave the rest away. Then he ate one.
   On the **third** day he decided to keep $\frac{5}{9}$ of his peaches. He gave the rest away. Then he ate one.
   On the **fourth** day he decided to keep $\frac{2}{7}$ of his peaches. He gave the rest away. Then he ate one.
   On the **fifth** day he decided to keep $\frac{2}{3}$ of his peaches. He gave the rest away. Then he ate one.
   How many peaches did the monkey have left at the end?
3. Write the following continued fraction expansions in the fraction form. No need to simplify!
   (a) $[1, 2, 4, 5]$
   (b) $[0, 9, 4, 3]$
   (c) $[1, 7, 3, 2]$
   (d) $[4, 7, 2]$
4. Solve for the rational numbers associated to the continued fraction expansions given in Question 3.

5. Solve for the continued fraction expansions of the reciprocals of the rational numbers you solved for in Question 4, what do you notice?
   Note: the reciprocal of a rational number $\frac{a}{b}$ *is* $\frac{b}{a}$.
6. Solve for the continued fraction expansions of the following rational numbers:
   (a) $\frac{49}{11}$
   (b) $\frac{423}{95}$
7. Solve for the irrational number associated with the following infinite continued fraction expansions.
   (a) $[3, 2, 3, ...] = [3, \overline{2}]$
   (b) $[1, 4, 1, ...] = [1, \overline{4}]$
8. Solve for the infinite continued fraction expansions of the following irrational numbers (try finding the pattern as early as possible for fun!).
   (a) $\sqrt{3}$
   (b) $\sqrt{5}$